ENGINEER ONWARD UPWARD ENGINEERONUP.COM

Name: ____________________

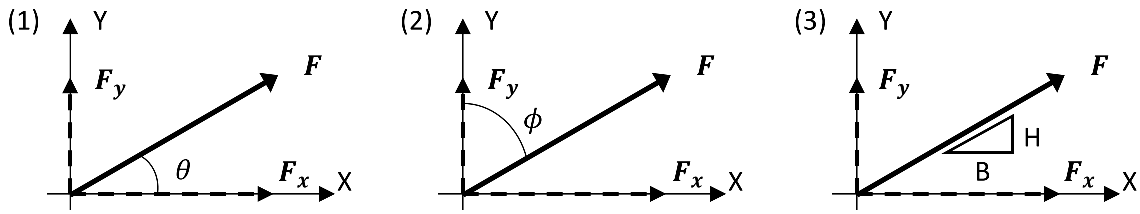
Course: CTC 224 Assignment: ____________

Description: CTC/MTC 224 Formula Sheet

Date: ____________

CTC/MTC 224 Formula Sheet for Statics and Strength of Materials

Force Components

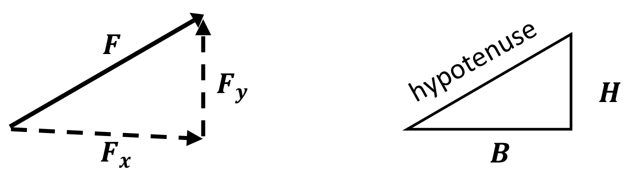


(1) $F_x = F\cos\theta$, $F_y = F\sin\theta$

(2) $F_x = F\sin\phi$, $F_y = F\cos\phi$

(3) $F_x = \frac{B}{hyp}F$, $F_y = \frac{H}{hyp}F$

Similar Triangles similar triangles have equal ratios between corresponding parts.



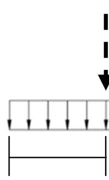
$$\frac{\boldsymbol{F_y}}{\boldsymbol{F_x}} = \frac{\boldsymbol{H}}{\boldsymbol{B}}$$

$$\boldsymbol{F} = \sqrt{\boldsymbol{F_x^2} + \boldsymbol{F_y^2}}$$

Distributed Forces

Uniform Loads

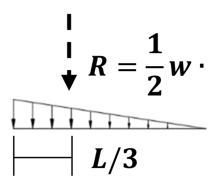
$\boldsymbol{R = w \cdot L}$



$\boldsymbol{L/2}$

Linear Loads

$\boldsymbol{R = \frac{1}{2} w \cdot L}$ ← Resultant



← Load

$\boldsymbol{L/3}$ ← Location of Resultant

Equations of Equilibrium

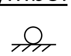
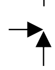
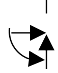
$\overset{+}{\rightarrow}\boldsymbol{\Sigma F_x = 0}$ The sum of **forces in the X-direction** equals zero; this is for HORIZONTAL force balance!

$+\uparrow \boldsymbol{\Sigma F_x = 0}$ The sum of **forces in the Y-direction** equals zero; this is for VERTICAL force balance!

$\circlearrowleft +\boldsymbol{\Sigma M_{pt} = 0}$ The sum of **moments** about any point equals zero; this is for ROTATIONAL force balance!
(A moment is a force times a perpendicular distance $M = F \cdot d_{\perp}$)

Beam Reactions

Step 1. Draw a Free-Body Diagram – show distributed force resultants, force at angle components, coordinate system, relevant dimensions (e.g. force distance from point A), and replace support symbols with force arrows.

Type	Symbol	Reactions	
Roller			1 force perpendicular to roller surface
Pin			2 forces at point of pin
Fixed			2 forces and 1 moment point of support

Step 2. Apply Equations of Equilibrium – typically solve:

1. $\Sigma M = 0$
2. $\Sigma F_y = 0$
3. $\Sigma F_x = 0$

Name: ____________________

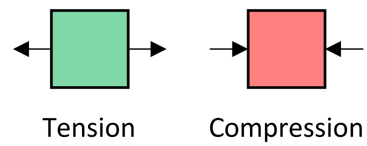
Course: CTC 224 Assignment: ____________

Description: CTC/MTC 224 Formula Sheet

Date: ____________

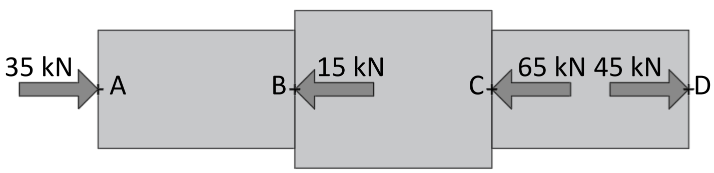
Axial Bar Forces

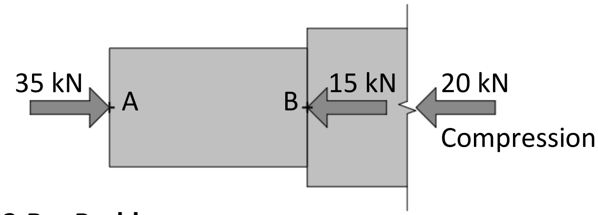
Axial forces align with the longitudinal axis of a member.
They create either tension (elongation) or compression (shortening).



1D Bar Problems

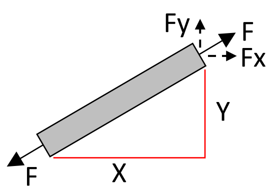
To find the force in a 1D bar: 1. “cut” the bar between its ends, 2. draw an FBD of either side, 3. apply $\Sigma F = 0$.





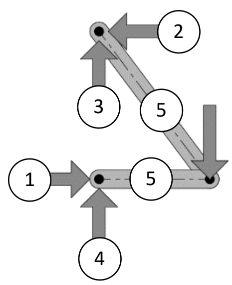
2-Force Members

Requirements:
1. Loaded at ends only
2. Pin-connected ends
3. Self-weight is negligible



**2-Force member forces are purely axial and the force in the member aligns with the member so that the ratio of forces is equal to the ratio of member geometry.*

$\frac{F_x}{F_y} = \frac{X}{Y}$ and $F = \sqrt{F_x^2 + F_y^2}$



2D 2-Bar Problems

To find the forces in 2D 2-bar problems - see left #

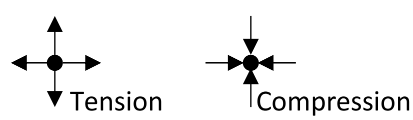
1. Apply $\Sigma M = 0$ to get one support reaction.
2. Apply $\Sigma F = 0$ to get a support reaction parallel to the reaction found in Step 1.
3. Use similar triangles to get a second reaction at one support so that the ratio of Fx:Fy matches the member’s geometry ratio X:Y
4. Apply $\Sigma F = 0$ to get last support reaction.
5. Use Pythagorean’s Theorem to get bar forces. Identify tension or compression.

Trusses

When solving trusses, assume unknown bar forces are in tension.

A **positive** (+) answer indicates the member force is in **tension**.

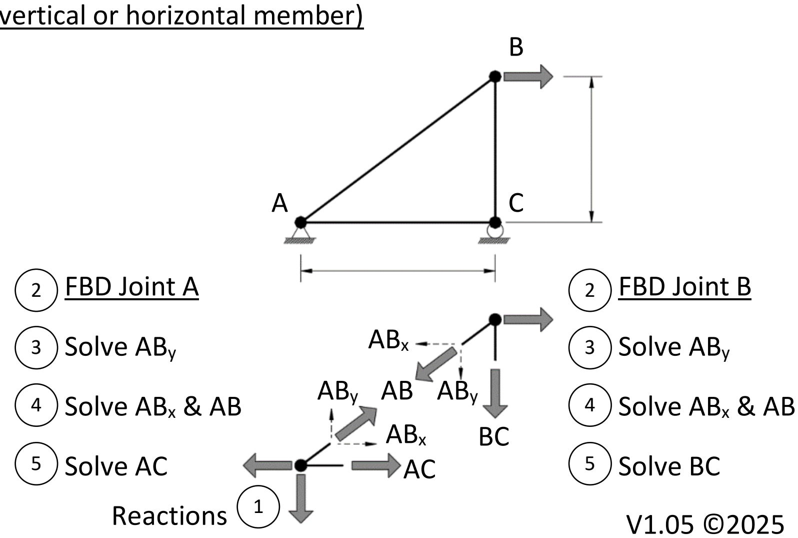
A **negative** (-) answer indicates the member force is in **compression**.



Method of Joints (for a joint with at least one vertical or horizontal member)

1. Solve for truss reactions (if needed).
2. Isolate a joint and draw an FBD.
3. Solve either $\boldsymbol{\Sigma F_x = 0}$ or $\boldsymbol{\Sigma F_y = 0}$ to obtain one force or component.
4. Apply trigonometry/similar triangles to solve for additional component and bar force (if needed).
5. Solve either $\boldsymbol{\Sigma F_y = 0}$ or $\boldsymbol{\Sigma F_x = 0}$ to obtain one force or component.

Tip: pick joints with only two unknown bar forces.



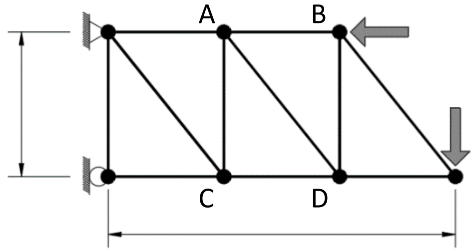
ENGINEER ONWARD UPWARD
ENGINEERONUP.COM

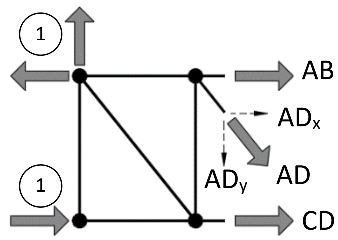
Name: ______________________
Course: CTC 224 Assignment: ______________
Description: CTC/MTC 224 Formula Sheet
Date: ______________

Trusses (cont'd)

Method of Sections (for parallel-chord trusses)

1. Solve for truss reactions (if needed).
2. Isolate a truss section and draw an FBD.
3. Solve either $\boldsymbol{\Sigma F_x = 0}$ and/or $\boldsymbol{\Sigma F_y = 0}$ to obtain one force or component.
4. Apply trigonometry/similar triangles to solve for additional component and bar force (if needed).
5. Solve $\boldsymbol{\Sigma M = 0}$ at the point of intersection of two unknown member forces or their components to solve for a bar force.
6. Repeat step 3. Or 5. to solve for last force.



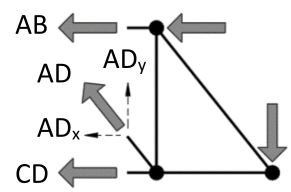


2 FBD Left Section
3 Solve AD_y
4 Solve AD_x & AD
5 Solve CD
6 Solve AB

Tip: you can also sum moments at a point not on the FBD.

Note: for non-parallel chord trusses, start by summing moments at a point of intersection of two unknown forces two times. Lastly, sum forces (or moments again) to solve for the remaining unknown bar force.

2 FBD Right Section
3 Solve AD_y
4 Solve AD_x & AD
5 Solve AB
6 Solve CD

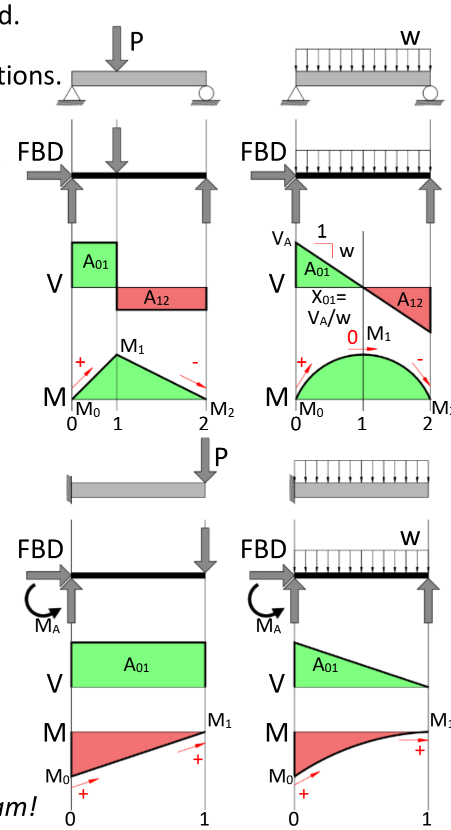


Load / Shear / Moment Diagrams

Load, shear and moment diagrams are often used to determine the maximum moment and shear in a beam.

To create load, shear and moment diagrams, the following steps can be used.

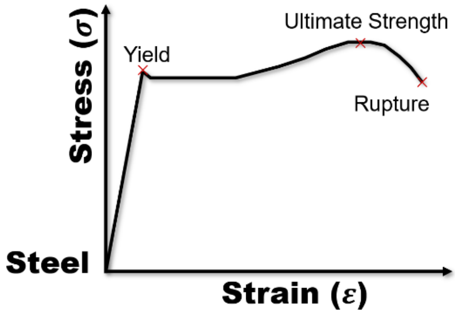
1. **Draw an FBD**, i.e., the load diagram of the beam, and solve for reactions.
   - Add vertical lines through reactions, point loads, critical points.
2. **Construct the shear diagram** by following the forces from left-right.
   - Point loads → vertical steps
   - Uniform loads → linear slope that matches the load
3. **Calculate shear areas** between lines in the V diagram (A_{01}, A_{12}, etc.)
   - For shear diagram triangles, x distance = shear / uniform load.
   - Also, add vertical line where V = 0.
   - Label all vertical lines.
   - Calculate areas of shear diagrams between sets of vertical lines.
4. **Calculate the moments** and **construct the moment diagram**.
   - $M_0 = 0$ for pin support w/o end moment, $M_0 = -M_a$ for cantilever.
   - $M_1 = M_0 + A_{01}$, $M_2 = M_1 + A_{12}$, etc.
   - Plot the moment and connect the dots.
     - No shear → horizontally sloped moment
     - Horizontal shear → linearly sloped moment
     - Linear sloped shear → parabolically curved moment

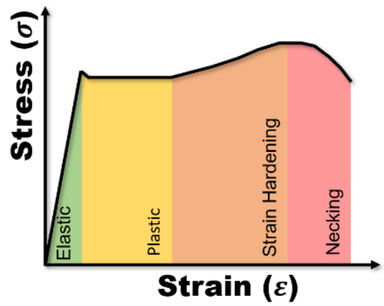


**The height of the shear diagram is equal to the slope of the moment diagram!*

Axial Stress, Strain, Deformation: *Force perpendicular to a cross-section causes axial stress in the cross-section.*

Axial stress $\sigma = \frac{P}{A}$ | Strain: $\varepsilon = \frac{\delta}{L}$ | Stress, strain & modulus of elasticity: $\sigma = \varepsilon E$ | Axial deformation $\delta = \frac{PL}{AE}$





Note: Representative stress-strain diagrams with critical values and zones labeled for a material like carbon-steel are shown to the left.

Shear Stress: *force parallel to a cross-section causes shear stress in the cross-section.*

Average shear stress: $\tau = \frac{V}{A}$ | where V is the shear stress and A is the shear area parallel to the shear force.

General shear stress: $\tau = \frac{VQ}{It}$ | Critical shear stress: $\tau_{rectangle} = \frac{3}{2} \cdot \frac{V}{A}$ | $\tau_{circle} = \frac{4}{3} \cdot \frac{V}{A}$ | $\tau_{wide\ flange} = \frac{V}{dt_w}$

Average shear stress in bolted systems: $\tau_{bolt} = \frac{V_{total}}{n_{bolts} \cdot n_{shear\ planes} \cdot A_{bolt}}$

Friction: $f = \mu F_N$

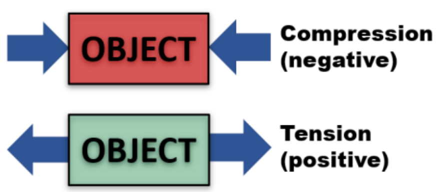
Bending Stress

Bending stress: $f_b = \frac{Mc}{I}$

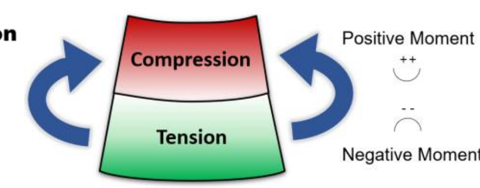
Combined Axial and Bending Stress

Combined Axial and Bending Stress: $f = \pm\sigma \pm f_b$

Use a sign convention where (+) is tension and (-) is compression.



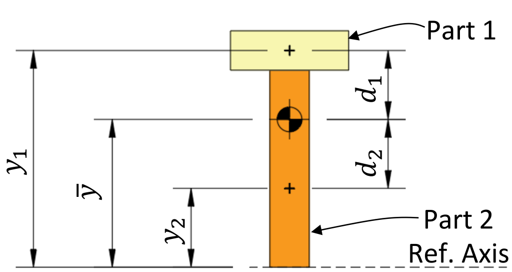
Axial Stress Sign Convention



Bending Stress Sign Convention

Moment of Inertia for basic standard shapes with horizontal symmetry: $I_{x_{rectangle}} = \frac{bh^3}{12}$ | $I_{x_{circle}} = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$

For complex shapes without horizontal symmetry, find the centroid $\bar{y} = \frac{\Sigma Ay}{\Sigma A}$ and use: $I_x = \Sigma(I_o + Ad^2)$.



Centroid Table

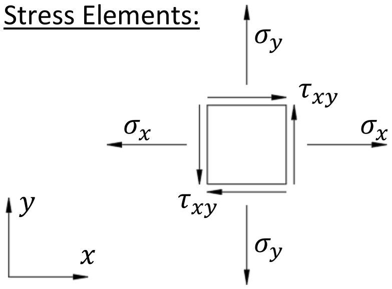
Part	A	y	Ay
1			
2			

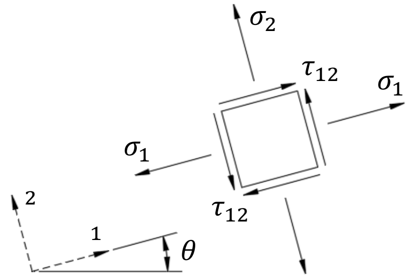
Moment of Inertia Table

Part	I_o	A	d	Ad^2
1				
2				

Stress Transformation

Stress Elements:



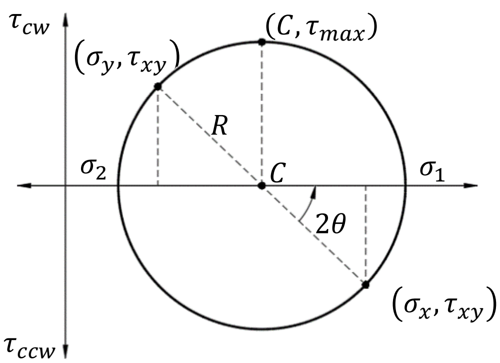


Stress Transformation Equations:

$$\sigma_1, \sigma_2 = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1-\sigma_2}{2}$$

Mohr's Circle



Mohr's Circle Equations:

$$C = \frac{\sigma_x+\sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = C + R$$

$$\sigma_2 = C - R$$

$$\tan 2\theta = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}$$

Notes:

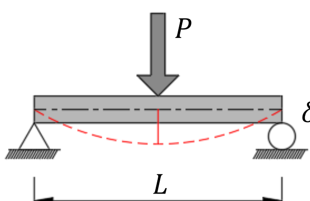
**The principal stresses σ_1 and σ_2 are found where the circle crosses the horizontal axis.*

**For axial stress, (+) is tension and (-) is compression*

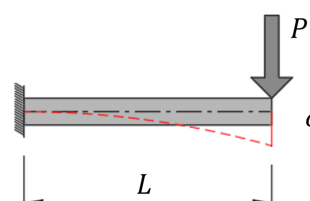
**The maximum shear stresses are found at the top and bottom of the circle.*

Beam Deflection while beam deflection can be solved using integration[4], for this class, use the formula method. Below are sample beam diagrams and formulas for common beam support and loading conditions.

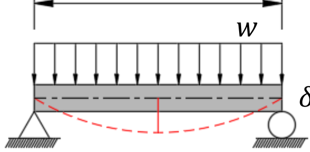
Deflection Formulas



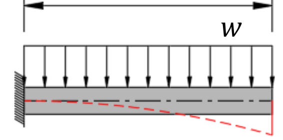
$$\delta_{max} = \frac{PL^3}{48EI}$$



$$\delta_{max} = \frac{PL^3}{3EI}$$

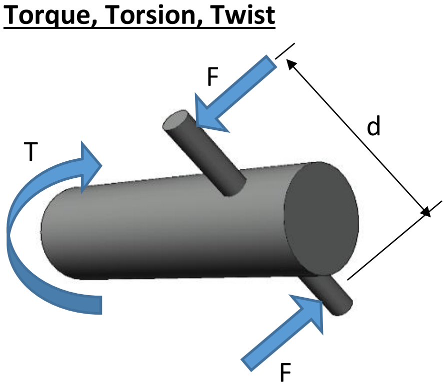


$$\delta_{max} = \frac{5wL^4}{384EI}$$



$$\delta_{max} = \frac{wL^4}{8EI}$$

Torque, Torsion, Twist



Torsion and Twist Equations

Torsional shear stress: $\tau = Tc/J$

for circles, $J = \frac{\pi}{32}d^4 = \frac{\pi}{2}r^4$, for d: diameter or r: radius

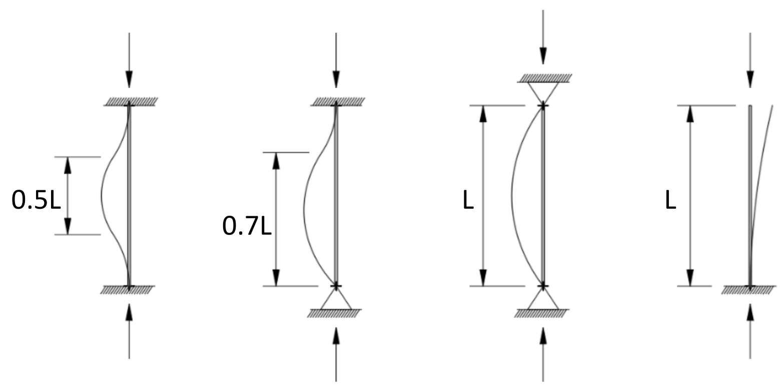
Twist of a round shaft is: $\theta = \frac{TL}{JG}$ in radians where π radians = 180°

ENGINEER ONWARD UPWARD
ENGINEERONUP.COM

Name: ______________________
Course: CTC 224 Assignment: ______________
Description: CTC/MTC 224 Formula Sheet
Date: ______________

Column Buckling

Material limit for compression: $P_y = F_y A$ | Euler's theoretical buckling *force*: $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$ | *stress*: $\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$



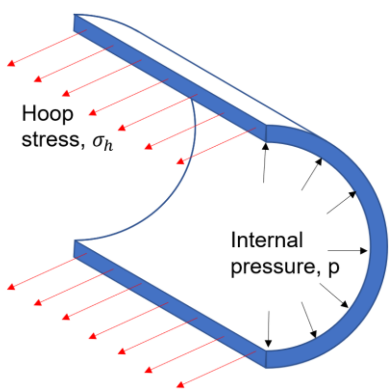
Note: for Fixed-free condition, only half of the "effective" buckled length is shown.

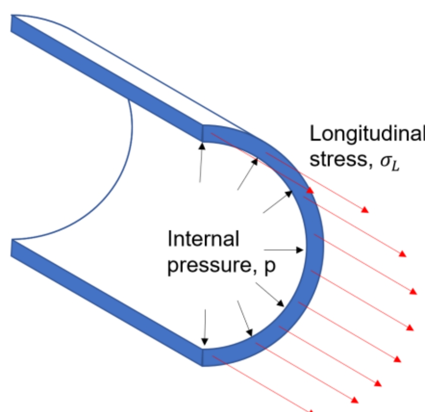
Theoretical K Value	0.5	0.7	1.0	2.0
End Conditions	Fixed-Fixed	Fixed-Pinned	Pinned-Pinned	Fixed-Free

Pressure Vessels Formulas below are for thin-walled pressure vessels, where $D > 20t$

Hoop stress is: $\sigma_h = \frac{pD}{2t}$

Longitudinal stress is: $\sigma_L = \frac{pD}{4t}$





Thermal Expansion and Thermal Stress

Change in length due to temperature change: $\delta = \alpha L(\Delta T)$ | Change in stress: $\sigma = \alpha E(\Delta T)$

Coefficient of thermal expansion for common engineering materials

Material	Coefficient of thermal expansion, α 10^{-6}/°F (10^{-6}/°C)
Steel	6.5 (11.7)
Aluminum	13.1 (23.6)
Polyvinyl Chloride	28.0 (50.4)

Factor of Safety

Factor of Safety is defined as the ratio of capacity to demand: $F.S. = \frac{capacity}{demand}$