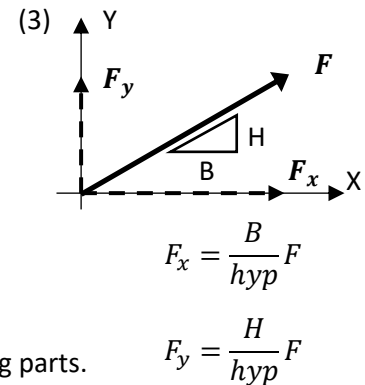
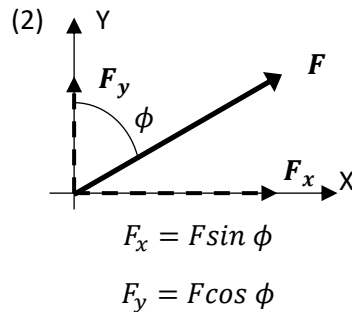
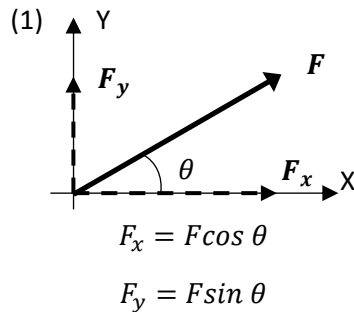
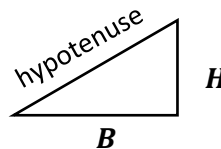
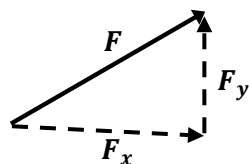


CTC/MTC 224 Formula Sheet for Statics and Strength of Materials

Force Components



Similar Triangles similar triangles have equal ratios between corresponding parts.

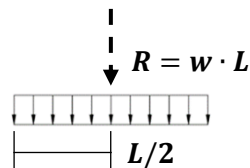


$$\frac{F_y}{F_x} = \frac{H}{B}$$

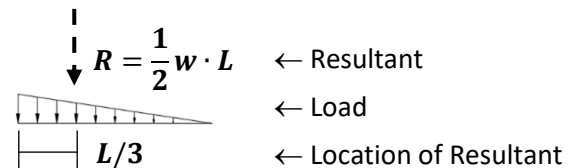
$$F = \sqrt{F_x^2 + F_y^2}$$

Distributed Forces

Uniform Loads



Linear Loads



Equations of Equilibrium

$\rightarrow \Sigma F_x = 0$ The sum of **forces in the X-direction** equals zero; this is for **HORIZONTAL** force balance!

$\uparrow \Sigma F_y = 0$ The sum of **forces in the Y-direction** equals zero; this is for **VERTICAL** force balance!

$\curvearrowright \Sigma M_{pt} = 0$ The sum of **moments** about any point equals zero; this is for **ROTATIONAL** force balance!
(A moment is a force times a perpendicular distance $M = F \cdot d_{\perp}$)

Beam Reactions

Step 1. Draw a Free-Body Diagram – show distributed force resultants, force at angle components, coordinate system, relevant dimensions (e.g. force distance from point A), and replace support symbols with force arrows.

Type	Symbol	Reactions
Roller		 1 force perpendicular to roller surface
Pin		 2 forces at point of pin
Fixed		 2 forces and 1 moment point of support

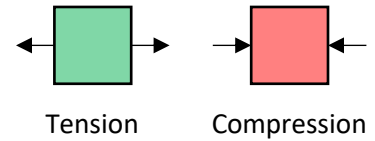
Step 2. Apply Equations of Equilibrium – typically solve:

- $\Sigma M = 0$
- $\Sigma F_y = 0$
- $\Sigma F_x = 0$

Axial Bar Forces

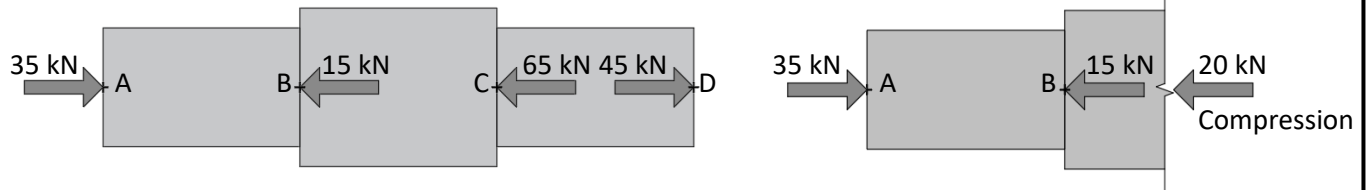
Axial forces align with the longitudinal axis of a member.

They create either tension (elongation) or compression (shortening).



1D Bar Problems

To find the force in a 1D bar: 1. "cut" the bar between its ends, 2. draw an FBD of either side, 3. apply $\Sigma F = 0$.



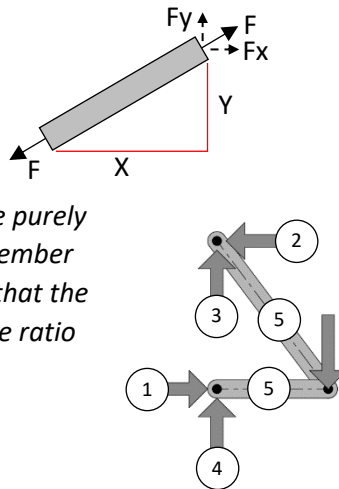
2-Force Members

Requirements:

1. Loaded at ends only
2. Pin-connected ends
3. Self-weight is negligible

**2-Force member forces are purely axial and the force in the member aligns with the member so that the ratio of forces is equal to the ratio of member geometry.*

$$\frac{F_x}{F_y} = \frac{X}{Y} \text{ and } F = \sqrt{F_x^2 + F_y^2}$$



2D 2-Bar Problems

To find the forces in 2D 2-bar problems - see left (#)

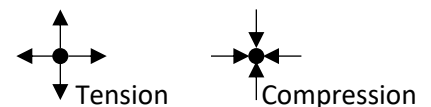
1. Apply $\Sigma M = 0$ to get one support reaction.
2. Apply $\Sigma F = 0$ to get a support reaction parallel to the reaction found in Step 1.
3. Use similar triangles to get a second reaction at one support so that the ratio of $F_x:F_y$ matches the member's geometry ratio $X:Y$.
4. Apply $\Sigma F = 0$ to get last support reaction.
5. Use Pythagorean's Theorem to get bar forces. Identify tension or compression.

Trusses

When solving trusses, assume unknown bar forces are in tension.

A **positive (+)** answer indicates the member force is in **tension**.

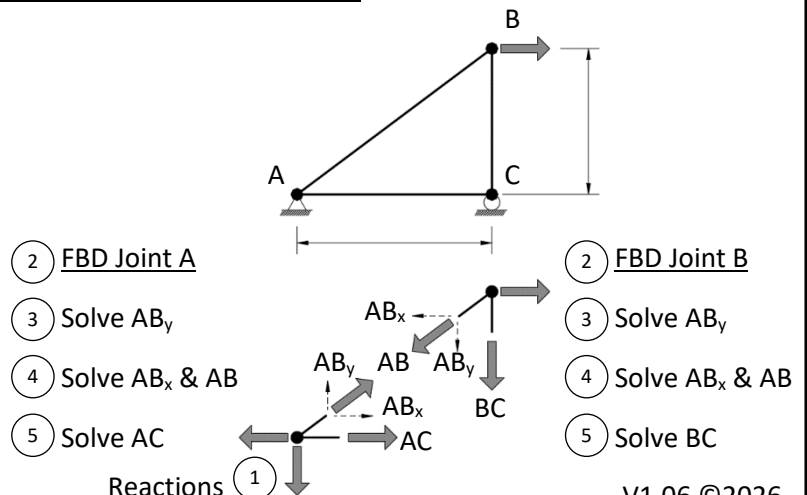
A **negative (-)** answer indicates the member force is in **compression**.



Method of Joints (for a joint with at least one vertical or horizontal member)

1. Solve for truss reactions (if needed).
2. Isolate a joint and draw an FBD.
3. Solve either $\Sigma F_x = 0$ or $\Sigma F_y = 0$ to obtain one force or component.
4. Apply trigonometry/similar triangles to solve for additional component and bar force (if needed).
5. Solve either $\Sigma F_y = 0$ or $\Sigma F_x = 0$ to obtain one force or component.

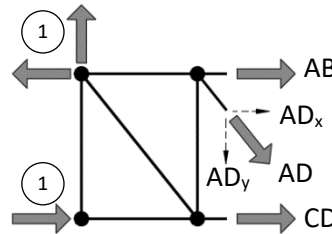
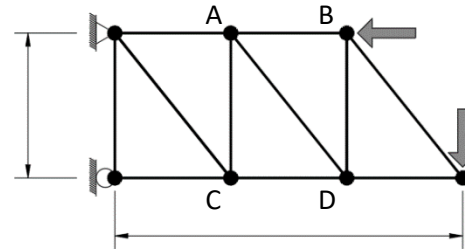
Tip: pick joints with only two unknown bar forces.



Trusses (cont'd)

Method of Sections (for parallel-chord trusses)

1. Solve for truss reactions (if needed).
2. Isolate a truss section and draw an FBD.
3. Solve either $\Sigma F_x = 0$ and/or $\Sigma F_y = 0$ to obtain one force or component.
4. Apply trigonometry/similar triangles to solve for additional component and bar force (if needed).
5. Solve $\Sigma M = 0$ at the point of intersection of two unknown member forces or their components to solve for a bar force.
6. Repeat step 3. Or 5. to solve for last force.



② FBD Left Section

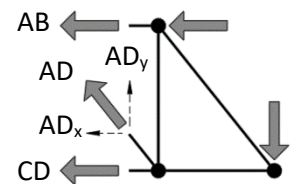
- ③ Solve AD_y
- ④ Solve AD_x & AD
- ⑤ Solve CD
- ⑥ Solve AB

Tip: you can also sum moments at a point not on the FBD.

Note: for non-parallel chord trusses, start by summing moments at a point of intersection of two unknown forces two times. Lastly, sum forces (or moments again) to solve for the remaining unknown bar force.

② FBD Right Section

- ③ Solve AD_y
- ④ Solve AD_x & AD
- ⑤ Solve AB
- ⑥ Solve CD



Load / Shear / Moment Diagrams

Load, shear and moment diagrams are often used to determine the maximum moment and shear in a beam.

To create load, shear and moment diagrams, the following steps can be used.

1. **Draw an FBD**, i.e., the load diagram of the beam, and solve for reactions.

- Add vertical lines through reactions, point loads, critical points.

2. **Construct the shear diagram** by following the forces from left-right.

- Point loads $\rightarrow$ vertical steps
- Uniform loads $\rightarrow$ linear slope that matches the load

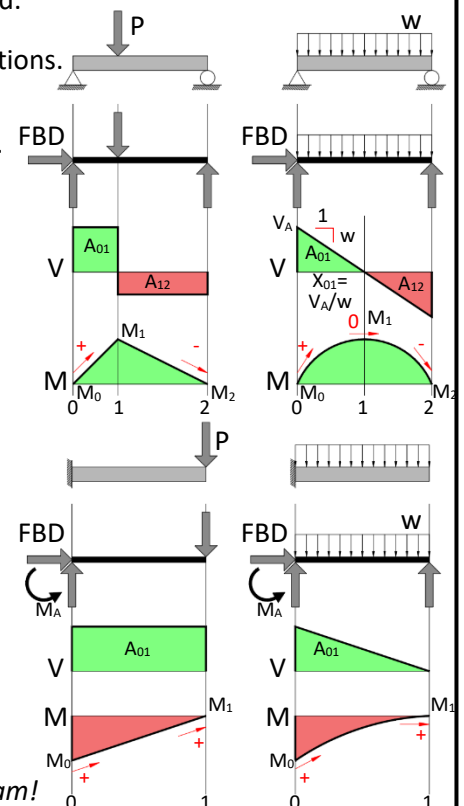
3. **Calculate shear areas** between lines in the V diagram (A_{01} , A_{12} , etc.)

- For shear diagram triangles, x distance = shear / uniform load.
- Also, add vertical line where $V = 0$.
- Label all vertical lines.
- Calculate areas of shear diagrams between sets of vertical lines.

4. **Calculate the moments and construct the moment diagram.**

- $M_0 = 0$ for pin support w/o end moment, $M_0 = -M_a$ for cantilever.
- $M_1 = M_0 + A_{01}$, $M_2 = M_1 + A_{12}$, etc.
- Plot the moment and connect the dots.
 - No shear $\rightarrow$ horizontally sloped moment
 - Horizontal shear $\rightarrow$ linearly sloped moment
 - Linear sloped shear $\rightarrow$ parabolically curved moment

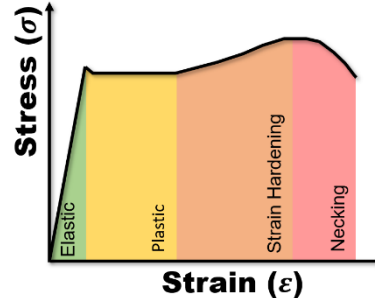
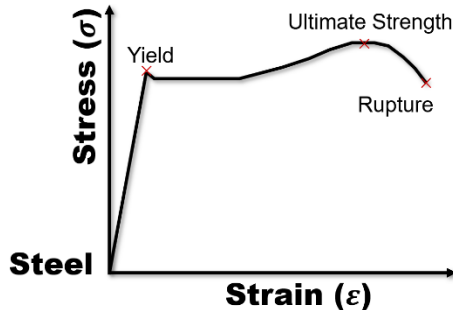
**The height of the shear diagram is equal to the slope of the moment diagram!*





Axial Stress, Strain, Deformation: Force perpendicular to a cross-section causes axial stress in the cross-section.

Axial stress $\sigma = \frac{P}{A}$ | Strain: $\varepsilon = \frac{\delta}{L}$ | Stress, strain & modulus of elasticity: $\sigma = \varepsilon E$ | Axial deformation $\delta = \frac{PL}{AE}$



Note: Representative stress-strain diagrams with critical values and zones labeled for a material like carbon-steel are shown to the left.

Shear Stress: force parallel to a cross-section causes shear stress in the cross-section.

Average shear stress: $\tau = \frac{V}{A}$ | where V is the shear stress and A is the shear area parallel to the shear force.

General shear stress: $\tau = \frac{VQ}{It}$ | Critical shear stress: $\tau_{rectangle} = \frac{3}{2} \cdot \frac{V}{A}$ | $\tau_{circle} = \frac{4}{3} \cdot \frac{V}{A}$ | $\tau_{wide\ flange} = \frac{V}{dt_w}$

Average shear stress in bolted systems: $\tau_{bolt} = \frac{V_{total}}{n_{bolts} \cdot n_{shear\ planes} \cdot A_{bolt}}$

Friction: $f = \mu F_N$

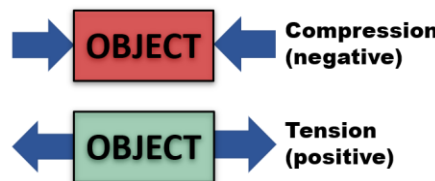
Bending Stress

Bending stress: $f_b = \frac{Mc}{I}$

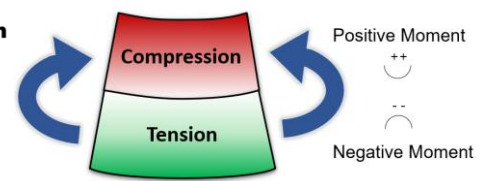
Combined Axial and Bending Stress

Combined Axial and Bending Stress: $f = \pm \sigma \pm f_b$

Use a sign convention where (+) is tension and (-) is compression.



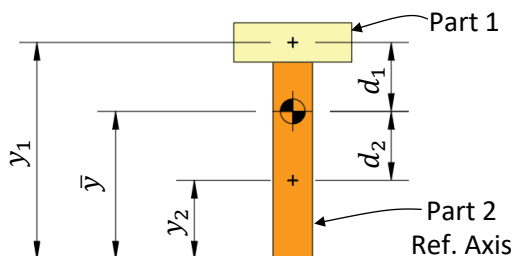
Axial Stress Sign Convention



Bending Stress Sign Convention

Moment of Inertia for basic standard shapes with horizontal symmetry: $I_{x_{rectangle}} = \frac{bh^3}{12}$ | $I_{x_{circle}} = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$

For complex shapes without horizontal symmetry, find the centroid $\bar{y} = \frac{\sum Ay}{\sum A}$ and use: $I_x = \sum(I_o + Ad^2)$.



Centroid Table

Part	A	y	Ay
1			
2			

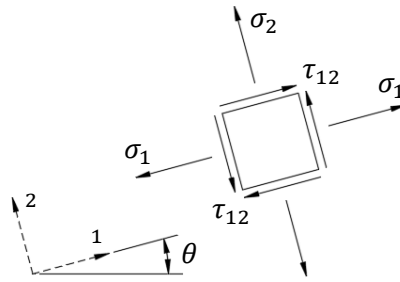
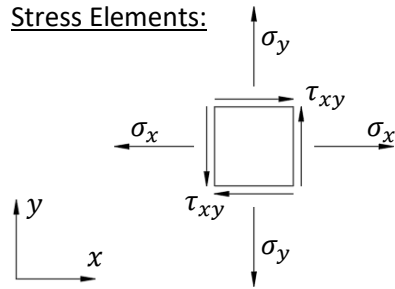
Moment of Inertia Table

Part	I_o	A	d	Ad^2
1				
2				



Stress Transformation

Stress Elements:

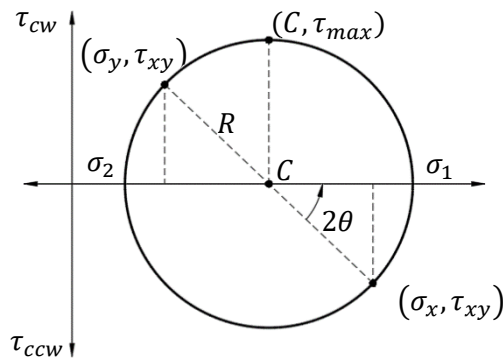


Stress Transformation Equations:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

Mohr's Circle



Mohr's Circle Equations:

$$C = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = C + R$$

$$\sigma_2 = C - R$$

$$\tan 2\theta = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

Notes:

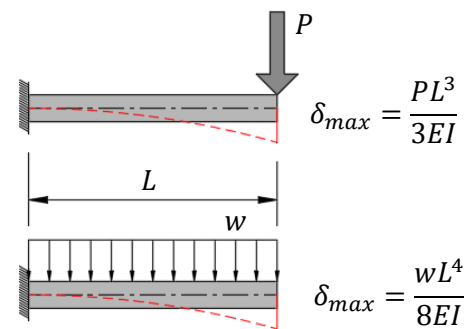
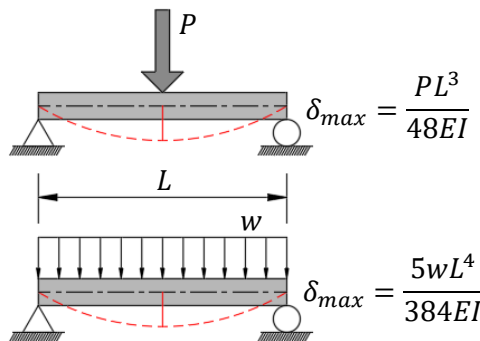
*The principal stresses σ_1 and σ_2 are found where the circle crosses the horizontal axis.

*For axial stress, (+) is tension and (-) is compression

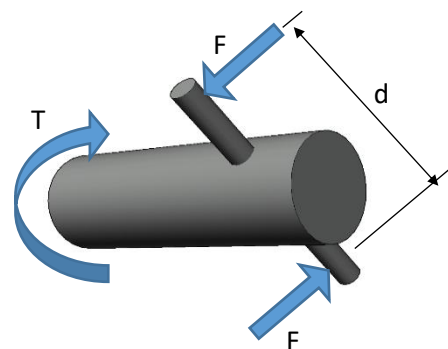
*The maximum shear stresses are found at the top and bottom of the circle.

Beam Deflection while beam deflection can be solved using integration⁴, for this class, use the formula method. Below are sample beam diagrams and formulas for common beam support and loading conditions.

Deflection Formulas



Torque, Torsion, Twist



Torsion and Twist Equations

$$\text{Torsional shear stress: } \tau = Tc/J$$

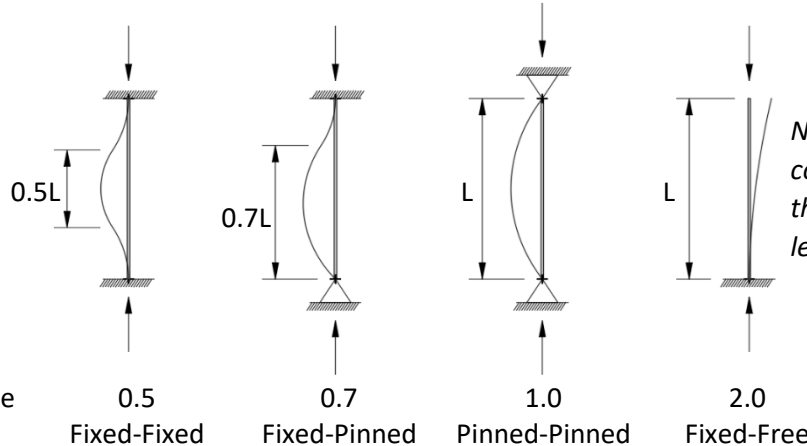
$$\text{for circles, } J = \frac{\pi}{32} d^4 = \frac{\pi}{2} r^4, \text{ for } d: \text{ diameter or } r: \text{ radius}$$

$$\text{Twist of a round shaft is: } \theta = \frac{TL}{JG} \text{ in radians where } \pi \text{ radians} = 180^\circ$$



Column Buckling

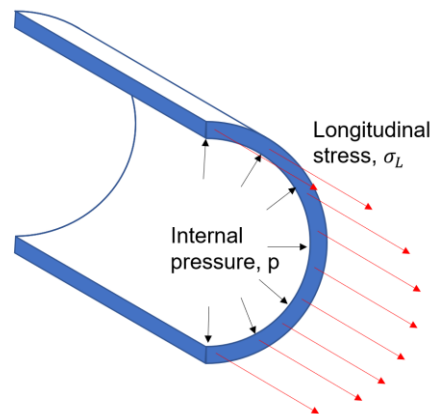
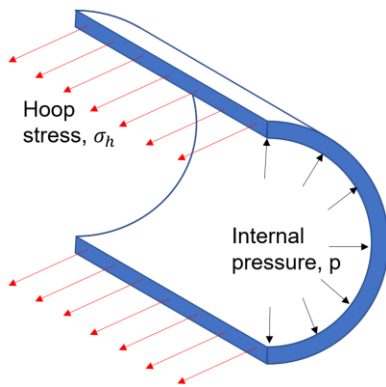
Material limit for compression: $P_y = F_y A$ | Euler's theoretical buckling force: $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$ | stress: $\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$



Pressure Vessels Formulas below are for thin-walled pressure vessels, where $D > 20t$

Hoop stress is: $\sigma_h = \frac{pD}{2t}$

Longitudinal stress is: $\sigma_L = \frac{pD}{4t}$



Thermal Expansion and Thermal Stress

Change in length due to temperature change: $\delta = \alpha L(\Delta T)$ | Change in stress: $\sigma = \alpha E(\Delta T)$

Coefficient of thermal expansion for common engineering materials

Material	Coefficient of thermal expansion, α $10^{-6}/^{\circ}\text{F}$ ($10^{-6}/^{\circ}\text{C}$)
Steel	6.5 (11.7)
Aluminum	13.1 (23.6)
Polyvinyl Chloride	28.0 (50.4)

Factor of Safety

Factor of Safety is defined as the ratio of capacity to demand: $F.S. = \frac{\text{capacity}}{\text{demand}}$